Electric Circuits

By

Prof. Dr. Mousa Abd-Allah

Dr. Ayman Yousef

Dr. Amr Ameen

Contents

References

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CHAPTER I

SINUSOIDS, PHASORS AND RESONANCE

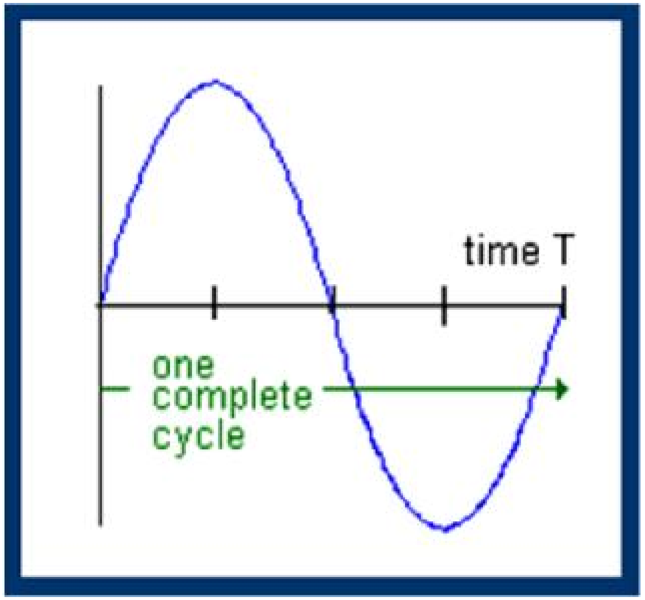
By

Dr. Ayman Yousef

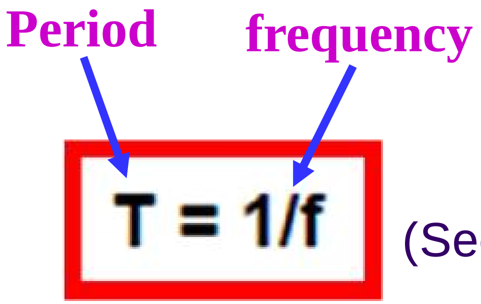
Sinusoids

Alternating (AC) Sinusoidal Waveforms

Period of a Waveform



- **Period (T):** The time interval between successive repetitions of a periodic waveform (time required for completing one full cycle).
- One period occupies exactly 360º of a sine waveform.



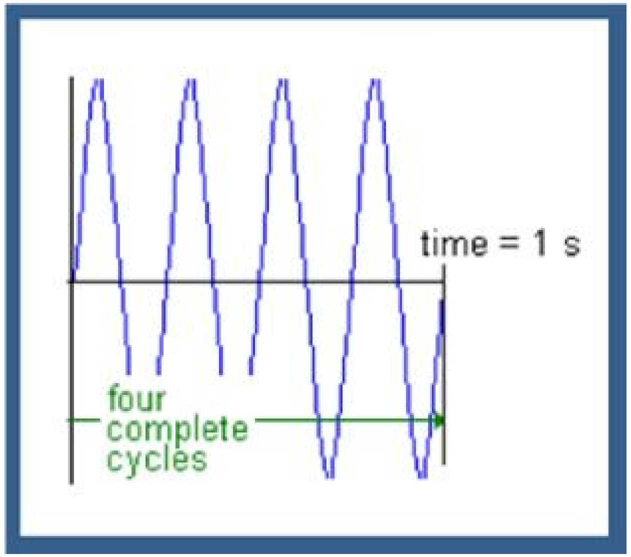
(Sec)

- **Cycle:** The portion of a waveform contained in one period of time.

Alternating (AC) Sinusoidal Waveforms

Frequency of a Waveform

- **Frequency (f):** The number of cycles that occur in 1 sec (number of cycles that is completed each second.)



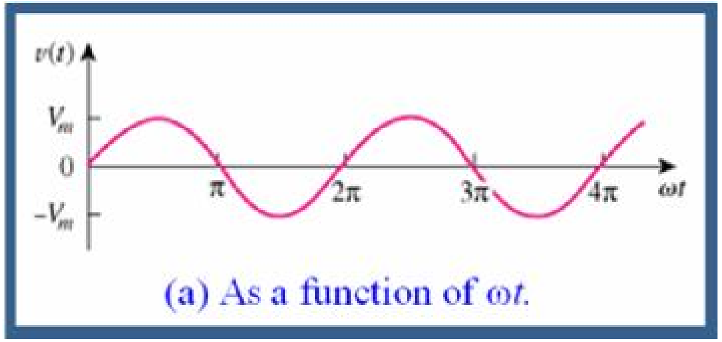
$$f = 1/T \quad \text{(Hertz)}$$

- This figure shows four cycles per second, or a waveform that has a frequency of 4 Hz.

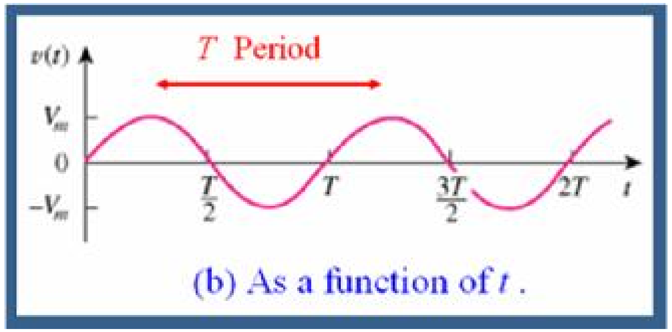
Sinusoids

- A **SINUSOID** is a signal that has the form of the sine or cosine function.

$$v(t) = V_m \sin \omega t$$



(a) As a function of ωt.



(b) As a function of t.

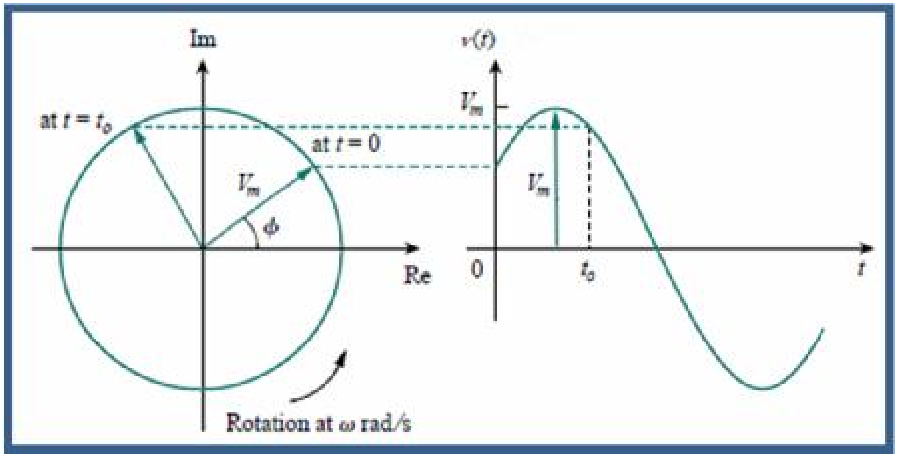
- V_m is the Peak Amplitude of the sinusoid.
- ω is the Angular Frequency (Angular Velocity) in radians/s.
- f is the Frequency in Hertz. • T is the period in seconds.

$\omega = 2\pi f$ and $f = 1/T$ ⇒ $T = \dfrac{2\pi}{\omega}$

Instantaneous Value of a Wave

Instantaneous value: The magnitude of a waveform at any instant of time; denoted by the lowercase letters ($v1$, $v2$).

Peak amplitude: The maximum value of the waveform as measured from reference horizontal line (the greater value of Instantaneous value), denoted by the uppercase letters Vm.



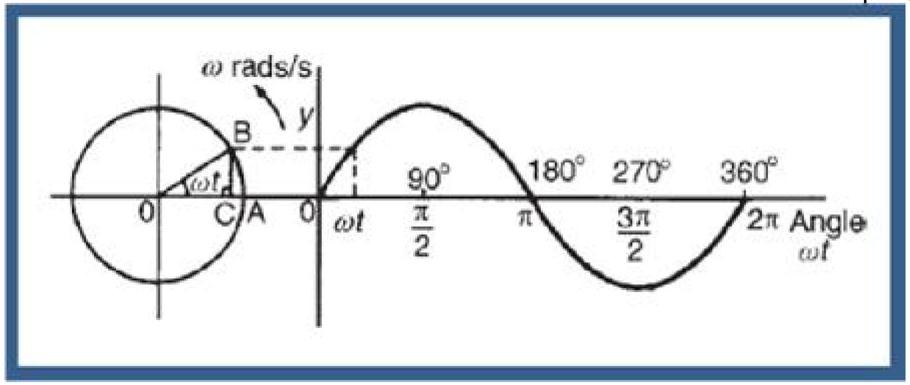
$$v(t)=V\text{m}\cos(\omega t+\varphi_v)$$

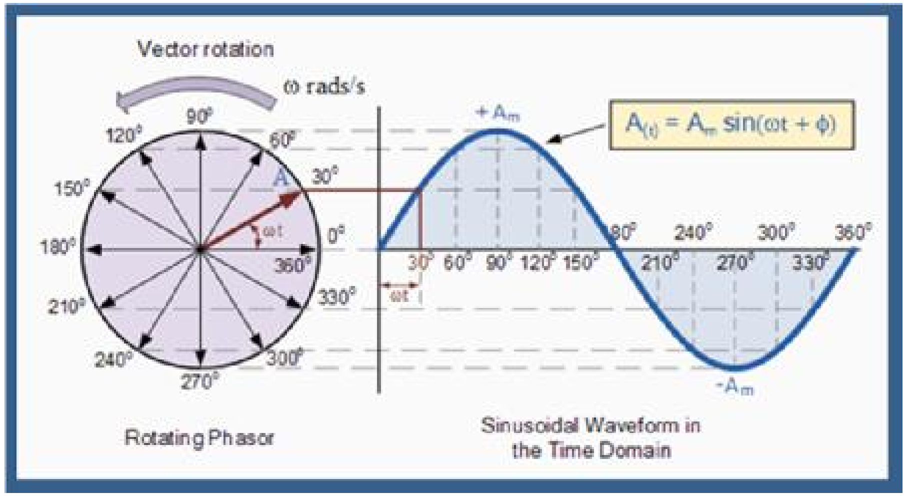
where φ_v **is the phase angle**

Phase of Sinusoids

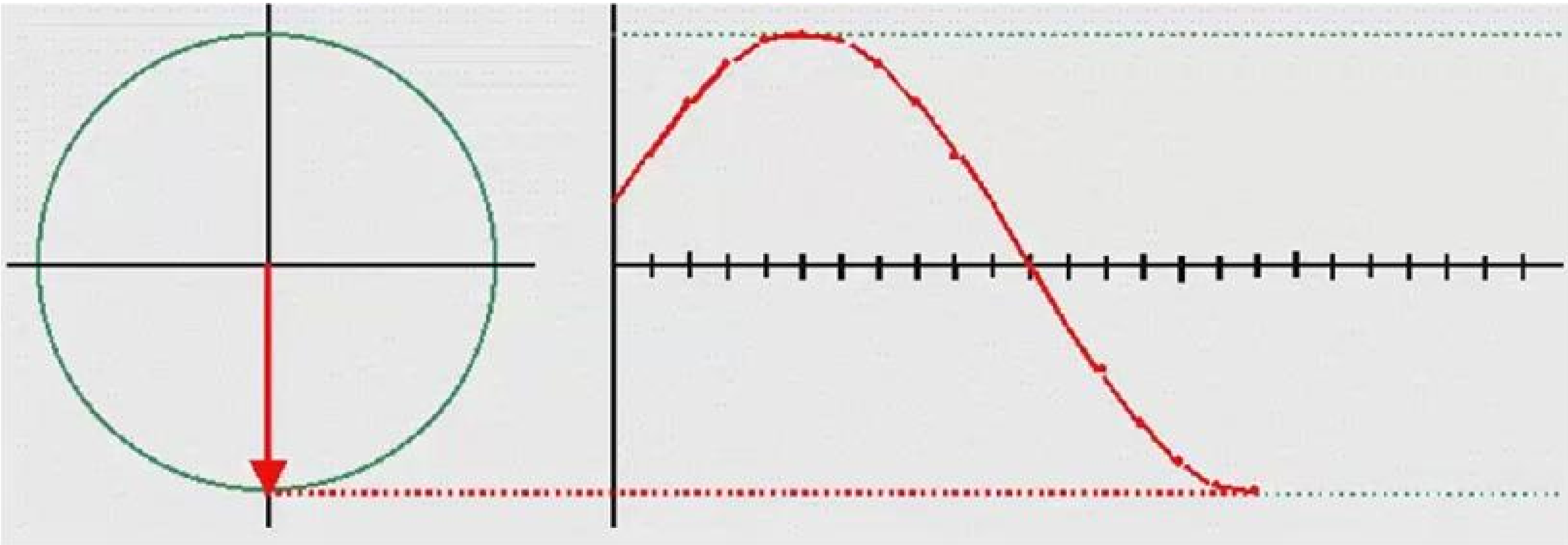
- OA represents a vector that is free to rotate anticlockwise about 0 at an angular velocity of ω rad/s. A rotating vector is known as a **phasor**.

- Any quantity which varies sinusoidally can thus be represented as a phasor.

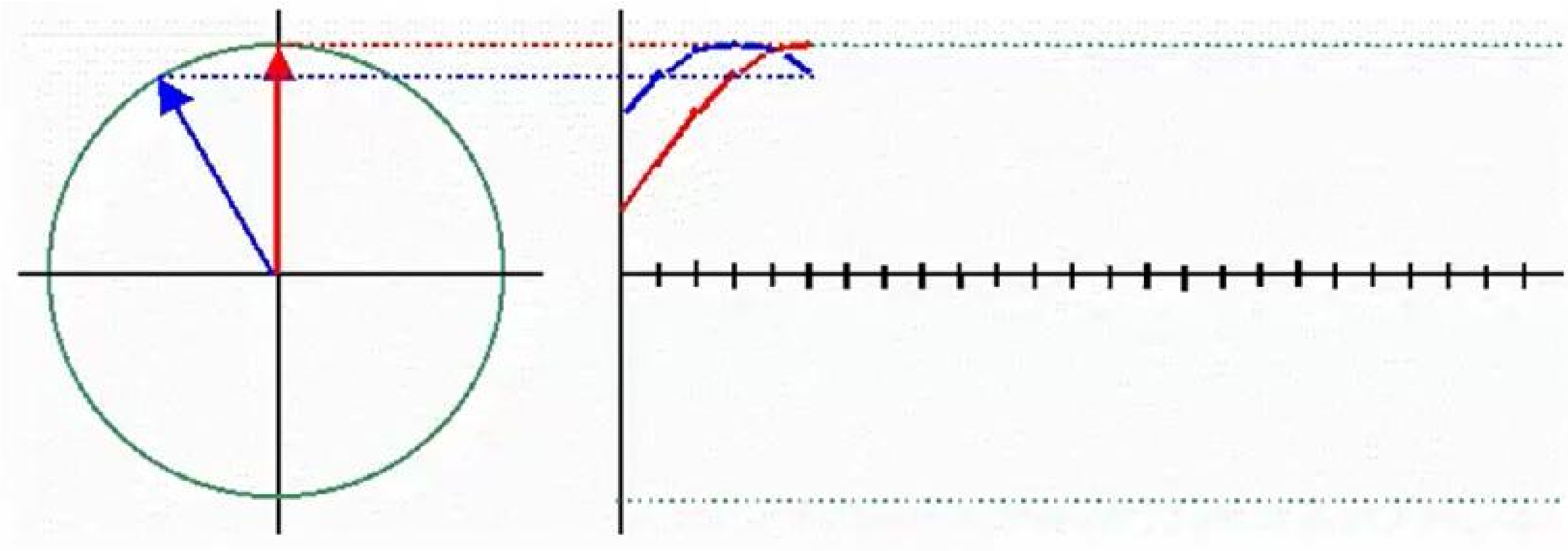




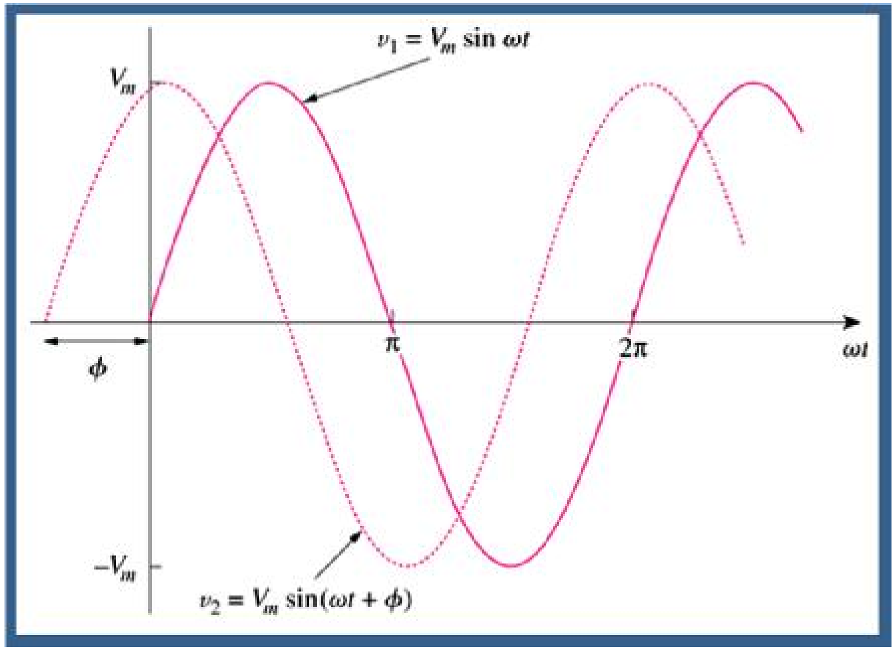
Rotating of single phasor



Rotating of Two phasors



Phase of Sinusoids

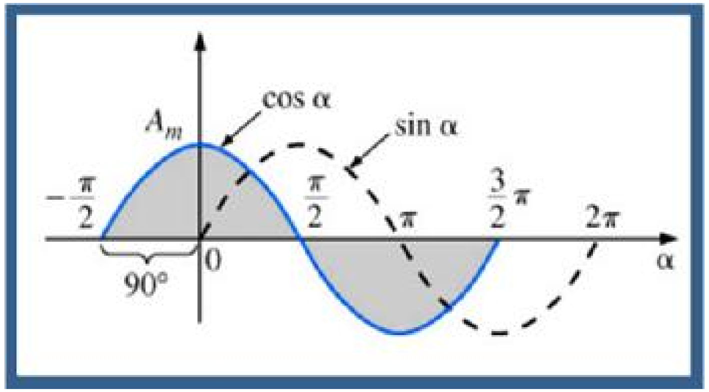


$$f = \frac{1}{T} Hz \quad \omega = 2\pi f$$

- **Only two sinusoidal values with the <u>same frequency</u> can be compared by their amplitude and phase difference.**
- **If phase difference is zero, they are in phase; if phase difference is not zero, they are out of phase.**

Phase of Sinusoids

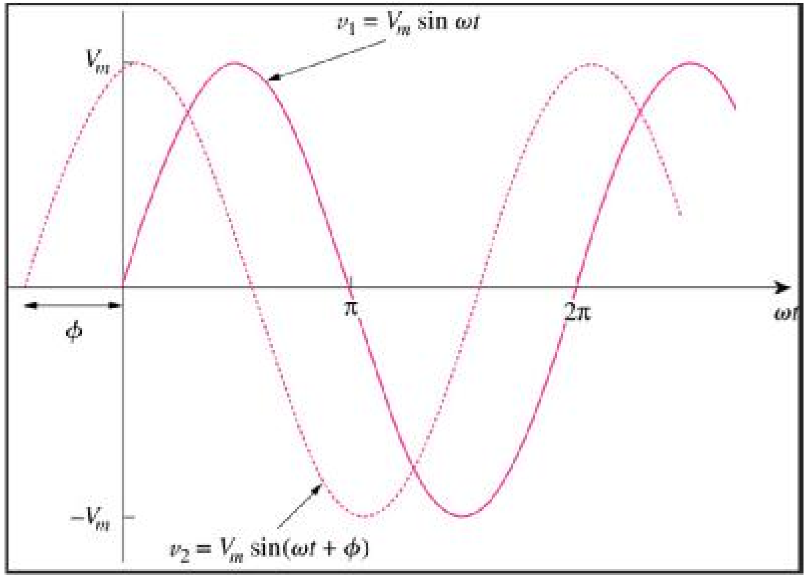
- The terms *lead* and *lag* are used to indicate the relationship between two sinusoidal waveforms of the *same frequency* plotted on the same set of axes.



- The cosine curve is said to *lead* the sine curve by 90°.
- The sine curve is said to *lag* the cosine curve by 90°.
- 90 is referred to as the phase angle between the two waveforms.
- When determining the phase measurement we first note that each sinusoidal function has the same frequency, permitting the use of either waveform to determine the period.
- Since the full period represents a cycle of 360°.

Phase of Sinusoids

- Consider the sinusoidal voltage having phase φ,



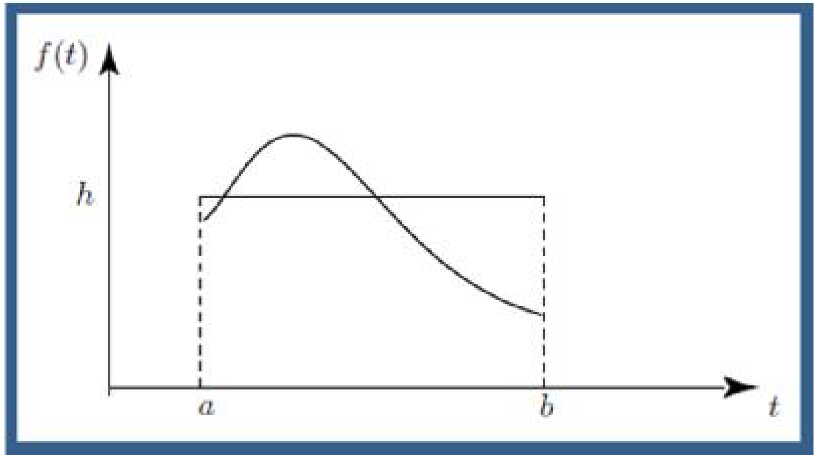
$$v_1(t) = V_m \sin \omega t$$

$$v_2(t) = V_m \sin(\omega t + \phi)$$

- v_2 LEADS v_1 by phase φ.
- v_1 LAGS v_2 by phase φ.
- v_1 and v_2 are out of phase.

Average Value of a Wave

Suppose a time-varying function $f(t)$ is defined on the interval $a \le t \le b$. The area A, under the graph of $f(t)$ is given by the integral



$$A = \int_a^b f(t)\mathrm{d}t$$

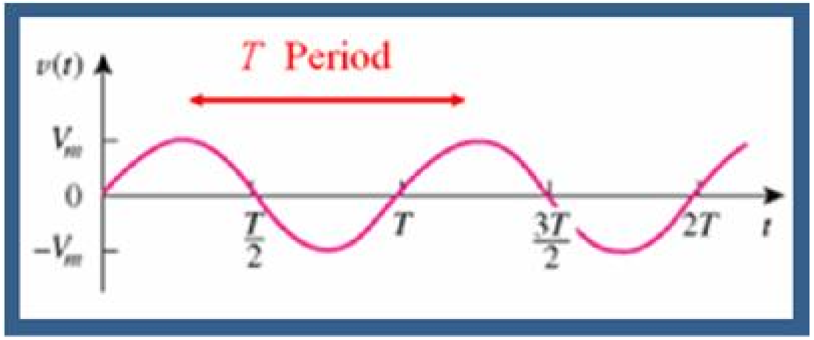
area of rectangle = area under curve

average or mean value of the function

$$h = \frac{1}{b-a}\int_a^b f(t)\mathrm{d}t$$

Average Value of Sine Wave

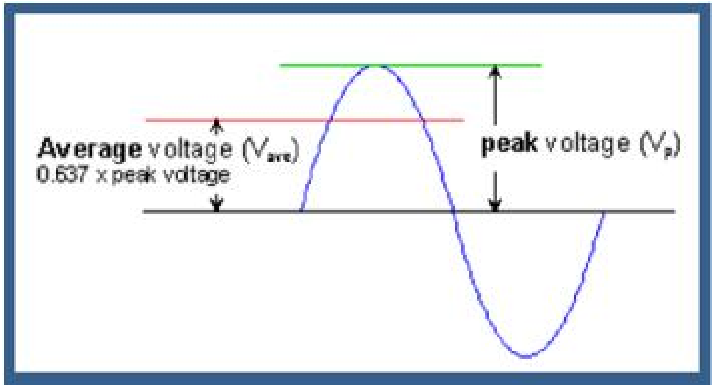
$$V_{av} = \frac{1}{T}\int_0^T v(t)dt$$



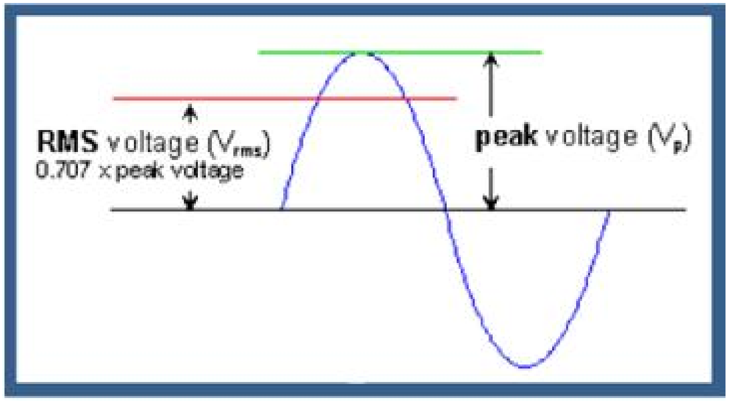
The average value of only half a cycle of the Sine wave is given by:

$$V_{av} = \frac{1}{T/2}\int_0^{T/2} v(t)dt = \frac{1}{T/2}\int_0^{T/2} V_m \sin(\omega t + \theta_v)dt = \frac{2}{\pi}V_m = 0.637V_m$$

The average value of the full cycle of the wave will be zero because the sine wave symmetrical about **zero.**



Root-mean-square value of sin wave



- The R.M.S. (or effective) value of an alternating voltage or current (AC) is the value which would produce the same amount of heat energy in a resistor as a direct voltage or current (DC) of the same magnitude.

R.M.S.

(3) Root

(2) Mean

(1) square

$$v(t) = V_m \sin \omega t \qquad V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} \qquad V_{rms} = 0.707\ V_m$$

Ex.1: **Find the amplitude, phase, period, and frequency of the sinusoid: $v(t)=12\cos(50t+10^o)$**

Solution

The amplitude is: $V_m = 12$ V

The phase is : $\phi = 10^o$

The angular frequency is $\omega = 50$ rad/s

The period is: $T = (2\pi/\omega) = (2\pi/50) = 0.1257$ s

The frequency is $f = (1/T) = (1/0.1257) = 7.958$ Hz

Ex.2 : Calculate the phase angle between
v_1 = -10 cos(wt+50º) & v_2 = 12 sin(wt-10º)
state which sinusoid is leading

Solution

$$v_1 = -10\cos(\omega t + 50^\circ) = 10\cos(\omega t + 50^\circ - 180^\circ)$$

$$v_1 = 10\cos(\omega t - 130^\circ)$$

$$v_2 = 12\sin(\omega t - 10^\circ) = 12\cos(\omega t - 10^\circ - 90^\circ)$$

$$v_2 = 12\cos(\omega t - 100^\circ)$$

v_2 leads v_1 by 30°

Complex Numbers

Complex Numbers

- A complex number may be written in RECTANGULAR FORM as:

RECTANGULAR FORM

$$z = x + jy \qquad j=\sqrt{-1},\ \ x=\mathrm{Re}(z),\ \ y=\mathrm{Im}(z)$$

- A second way of representing the complex number is by specifying the MAGNITUDE (*r*) and the ANGLE (*θ*) in POLAR form.

POLAR FORM

$$z = x + jy = |z| \angle \theta = r \angle \theta$$

- The third way of representing the complex number is the EXPONENTIAL form.

EXPONENTIAL FORM

$$z = x + jy = |z| \angle \theta = re^{j\theta}$$

- x is the REAL part.
- y is the IMAGINARY part.
- r is the MAGNITUDE.
- φ is the ANGLE.

Complex Numbers

- A complex number may be written in RECTANGULAR FORM as:

$z = x + jy \quad j = \sqrt{-1}$ RECTANGULAR FORM

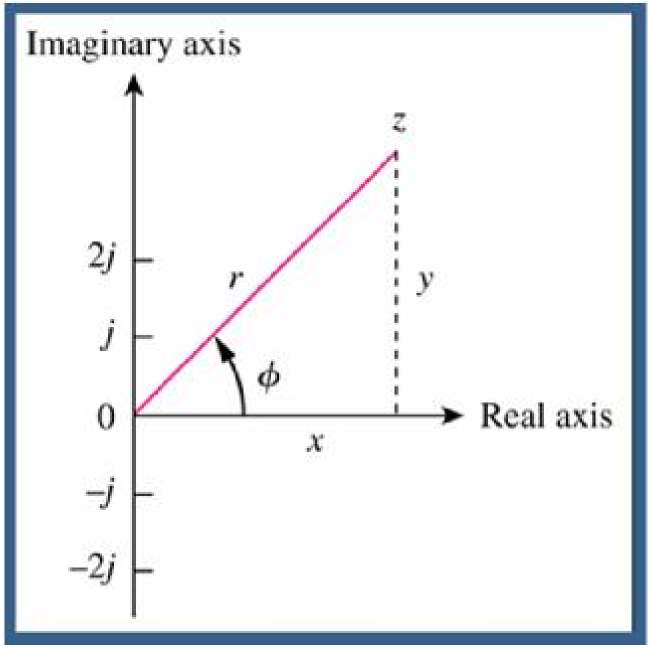
$x = r\cos\theta \qquad y = r\sin\theta$

$z = r\angle\phi$ POLAR FORM

$r = \sqrt{x^2 + y^2} \qquad \theta = \tan^{-1}\dfrac{y}{x}$

$z = re^{j\phi}$ EXPONENTIAL FORM

$r = \sqrt{x^2 + y^2} \qquad \theta = \tan^{-1}\dfrac{y}{x}$



$$z = x + jy = r\angle\phi = re^{j\phi}$$

Complex Number Conversions

- Mathematical operations on complex numbers may require conversions from one form to other form.

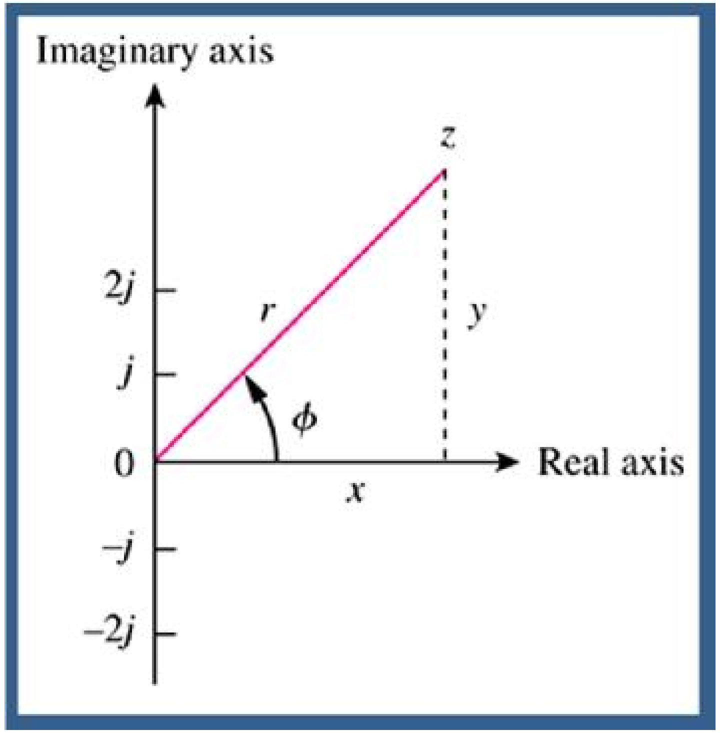
$$z = x + jy = r\angle\phi = re^{j\phi} = r(\cos\phi + j\sin\phi)$$

- Converting Rectangular to Polar:

$$z = x + jy$$

$$r = \sqrt{x^2 + y^2}, \quad \phi = \tan^{-1}\frac{y}{x}$$

$$z = r\angle\phi$$



- Converting Polar to Rectangular :

$$z = r\angle\phi$$

$$x = r\cos\phi, \quad y = r\sin\phi$$

$$z = x + jy$$

Mathematical Operations of Complex Numbers

- Mathematical operations for conversions complex numbers from one form to other form.

  - **Addition** and **Subtraction** must be on the **Rectangular** form.

$$\text{ADDITION: } z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

$$\text{SUBTRACTION: } z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

  - **Multiplication** and **Division** must be on the **Polar** form.

$$\text{MULTIPLICATION: } z_1 z_2 = r_1\, r_2 \angle \phi_1 + \phi_2$$

$$\text{DIVISION: } \frac{z_1}{z_2} = \frac{r_1}{r_2} \angle \phi_1 - \phi_2$$

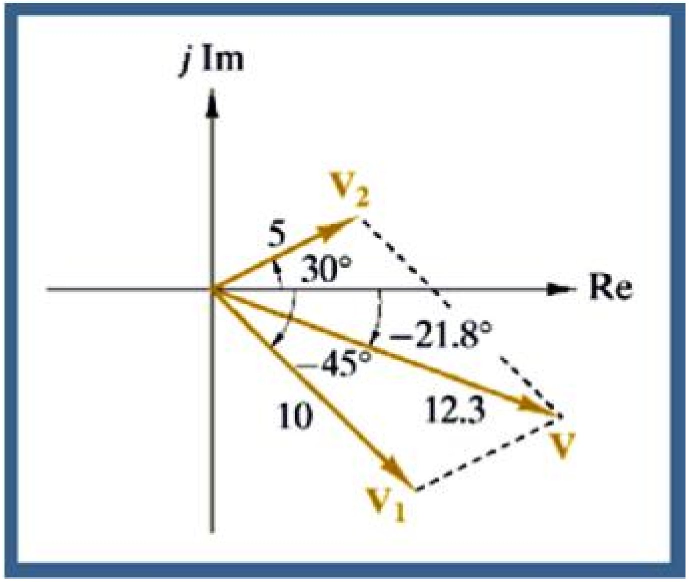
Adding Phasors Graphically

- Adding sinusoids of the same frequency is equivalent to adding their corresponding phasors.

$$\mathbf{V}=\mathbf{V}_1+\mathbf{V}_2$$

$$V_1 = V_1\angle -45^o \text{ v}$$

$$V_2 = V_2\angle 30^o \text{ v}$$



Phasors

Phasors

- The **phasor** is a complex number that carries the **amplitude** and **phase angle** information of a sinusoidal function.

- Phasor is the mathematical equivalent of a sinusoid with time variable dropped.

- Phasor will be defined from the **cosine function** in all our proceeding study.

- If a voltage or current expression is in the form of a sine, it will be changed to a cosine by subtracting from the phase.

Phasors

- Phasor representation is based on **Euler's formula**.

- **Euler's formula** indicates that sinusoids can be represented mathematically by the sum of two complex-valued functions

$$e^{\pm j\phi} = \cos\phi \pm j\sin\phi \quad \text{Euler's Identity}$$

$$\cos\phi = \text{Re}\{e^{j\phi}\} \quad \text{Real part} \qquad \sin\phi = \text{Im}\{e^{j\phi}\} \quad \text{Imaginary part}$$

Given a sinusoid $v(t)=V_m\cos(\omega t+\varphi)$

$$v(t) = V_m \cos(\omega t + \phi) = \text{Re}(V_m e^{j(\omega t+\phi)}) = \text{Re}(V_m e^{j\phi} e^{j\omega t}) = \text{Re}(\mathbf{V} e^{j\omega t})$$

$$\mathbf{V} = V_m e^{j\phi} = V_m \angle\phi = \text{PHASOR REP.}$$

Phasors

$v(t) = V_m \cos(\omega t + \phi)$ → (Time Domain Representation)

$\mathbf{V} = V_m \angle \phi$ → (Phasor Domain Representation)

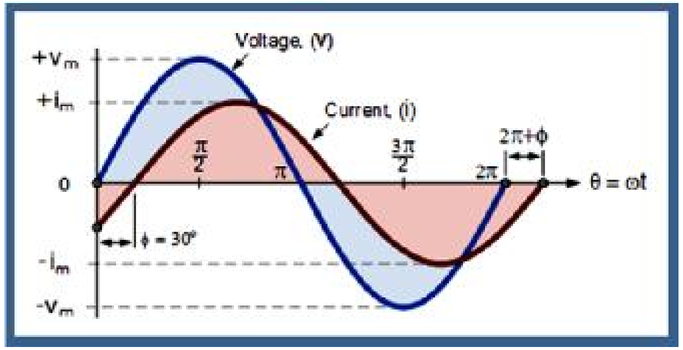
$v(t) = \mathrm{Re}\{\mathbf{V} e^{j\omega t}\}$ (Converting Phasor back to time)

- Given the sinusoids $i(t)=I_m\cos(\omega t+\varphi_I)$ and $v(t)=V_m\cos(\omega t+ \varphi_V)$ we can obtain the phasor forms as:

If $i(t) = I_m\cos(\omega t+\phi_I)$ then phasor $\hat{I} = I_m \angle \phi_I$

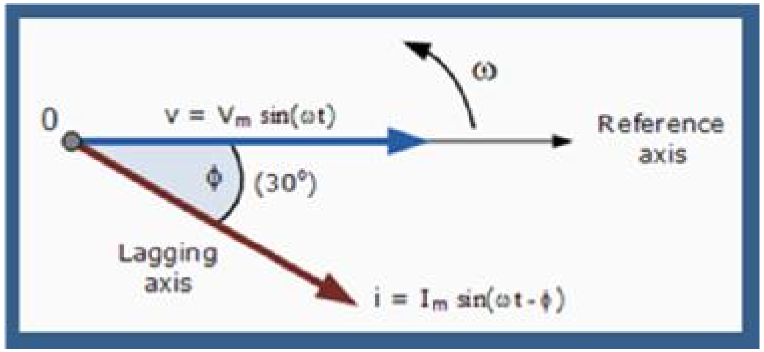
If $v(t) = V_m\cos(\omega t+\phi_V)$ then phasor $\hat{V} = V_m \angle \phi_V$

Phase Difference of a Sinusoidal Waveform



- The current, i is **lagging** the voltage, v by angle Φ and in our example above this is 30º.

- So the difference between the two phasors representing the two sinusoidal quantities is angle Φ and the resulting **phasor diagram** will be.



Ex. 3 : Transform these sinusoids to phasors:

(a) $i = 6\cos(50t - 40^o)$

(b) $v = -4\sin(30t + 50^o)$

Solution

(a) $i = 6\cos(50t - 40^o)$

The phasor	$I = 6\angle{-40^o}$

(b) $v = -4\sin(30t + 50^o)$

$= 4\cos(30t + 50 + 90^o)$

$= 4\cos(30t + 140^o)$

The phasor form of v is	$V = 4\angle 140^o$

Ex.4: Given $i_1(t)=4\cos(wt+30^o)$ A, and $i_2(t)=5\sin(wt-20^o)$ A , find their sum.

Solution

$i_1(t)= 4\cos(\omega t+30^o)$ then phasor $I_1 = 4\angle 30^o$ A

$i_2(t)= 5\sin(\omega t-20^o) = 5\cos(\omega t -20^o-90^o) = 5\cos(\omega t-110^o)$

then phasor $I_2 = 5\angle -110^o$ A

$$I = I_1 + I_2 = 4\angle 30^o + 5\angle -110^o = 3.464+j2-1.71-j4.698$$

$$= 1.754 - j2.698$$

$$I = I_1 + I_2 = 3.218\angle -56.97^o$$

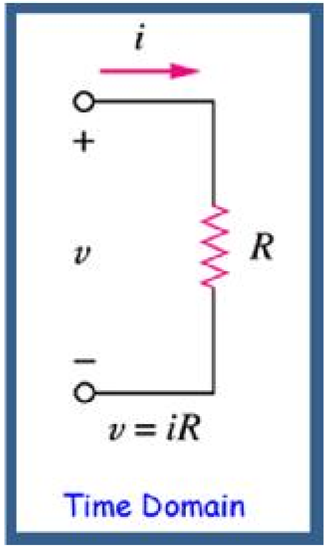
Phasor Relationships for Circuit Elements

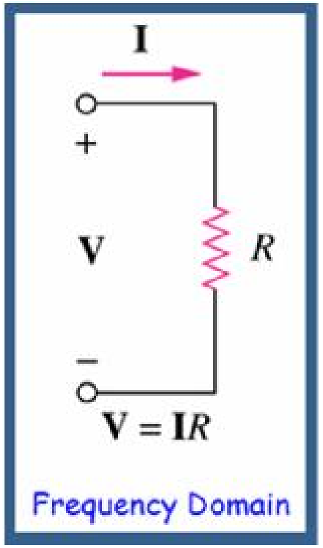
Phasor Relationships for Resistor

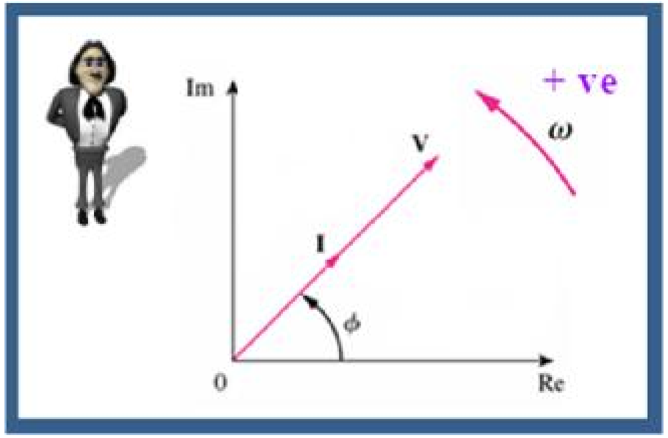
$$i(t) = I_m \cos(\omega t + \phi) = \mathrm{Re}(\mathbf{I} e^{j\omega t})$$

$$v(t) = i(t) R = R I_m \cos(\omega t + \phi)$$

$$\mathbf{V} = R I_m \angle \phi = R\mathbf{I}$$

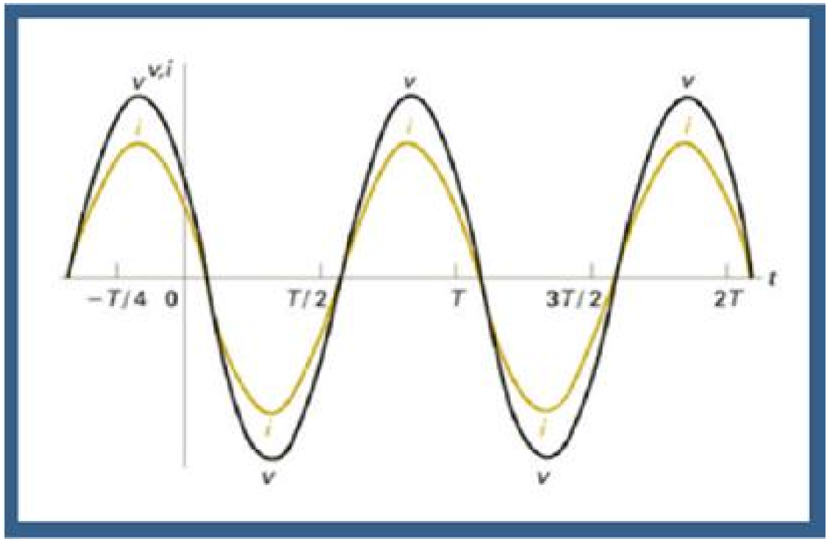


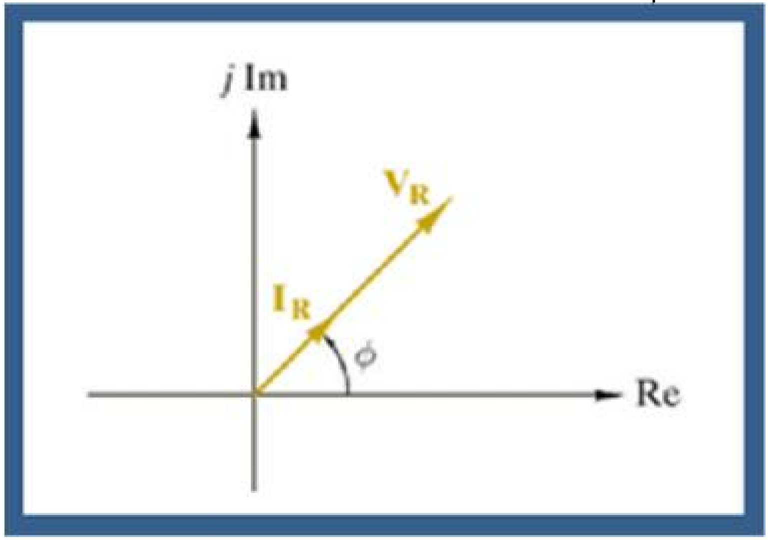




Phasor voltage and current of a resistor are in phase

Phasor Relationships for Resistor





Time Domain

Frequency Domain

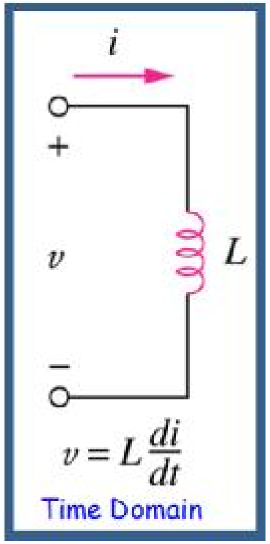
Phasor voltage and current of a resistor are in phase

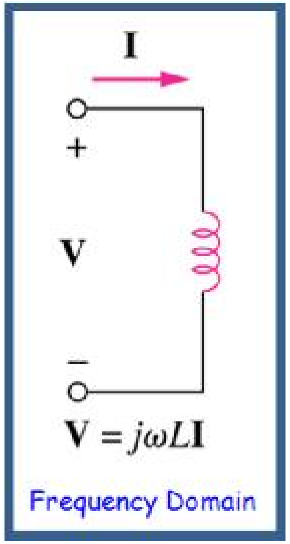
Phasor Relationships for Inductor

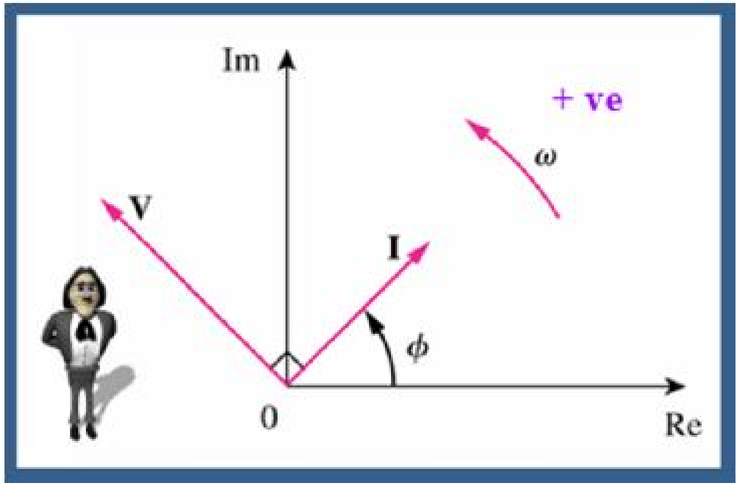
$$v(t) = L\frac{di}{dt} = L\frac{d}{dt}I_m\cos(\omega t+\phi) = -\omega L I_m\sin(\omega t+\phi) = \omega L I_m\cos(\omega t+\phi+90°)$$

$$\mathbf{V} = \omega L I_m \angle(\phi+90°) = \omega L I_m e^{j\phi}e^{j90°} = j\omega L\mathbf{I}$$

Where the Inductive reactance is: $X_L = \omega L$

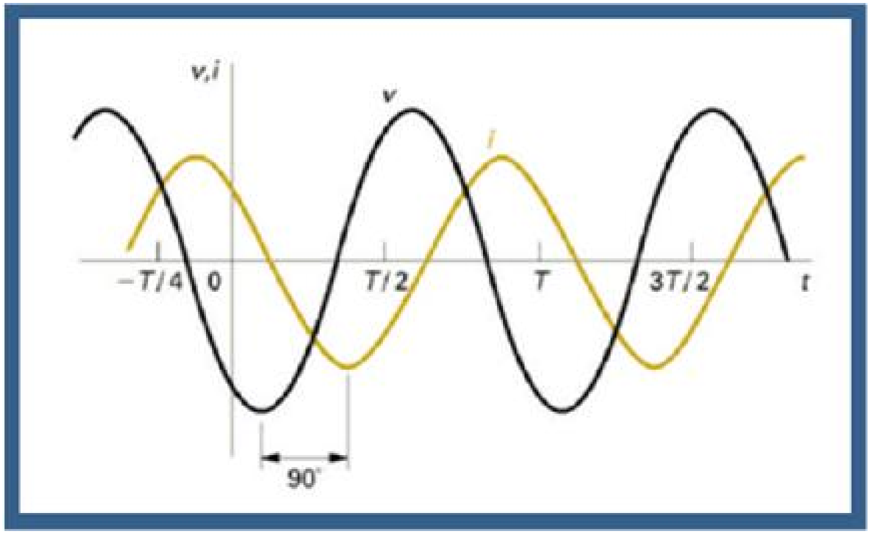




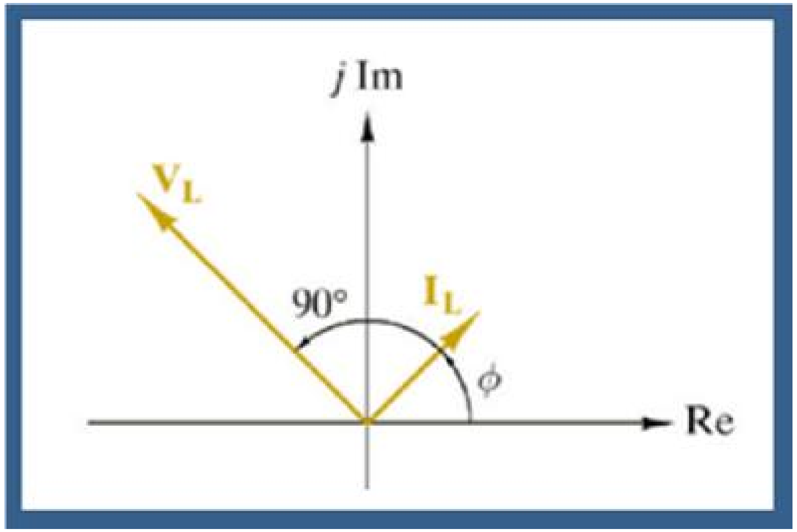


Phasor current of an inductor LAGS the voltage by 90 degrees.

Phasor Relationships for Inductor



Time Domain



Frequency Domain

Phasor current of an inductor LAGS the voltage by 90 degrees.

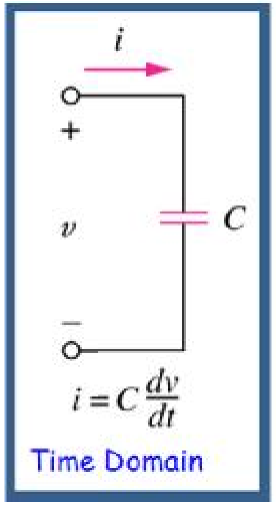
Phasor Relationships for Capacitor

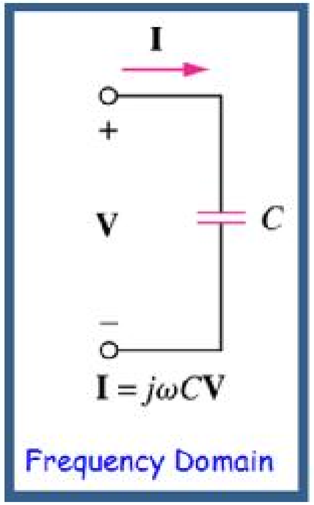
$$i(t) = C\frac{dv}{dt} = C\frac{d}{dt}V_m\cos(\omega t+\phi) = -\omega C V_m \sin(\omega t+\phi) = \omega C V_m\cos(\omega t+\phi+90°)$$

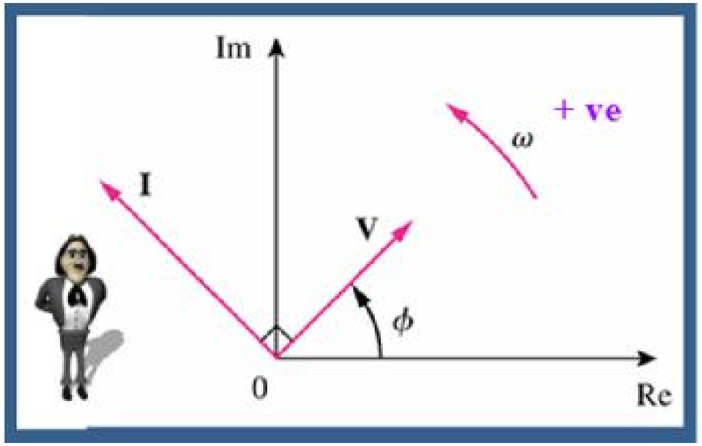
$$\mathbf{I} = \omega C V_m \angle(\phi+90°) = \omega C V_m e^{j\phi}e^{j90°} = j\omega C\mathbf{V} \qquad \mathbf{V} = \frac{\mathbf{I}}{j\omega C}$$

Where the capacitive reactance is:

$$X_C = \frac{1}{\omega C}$$

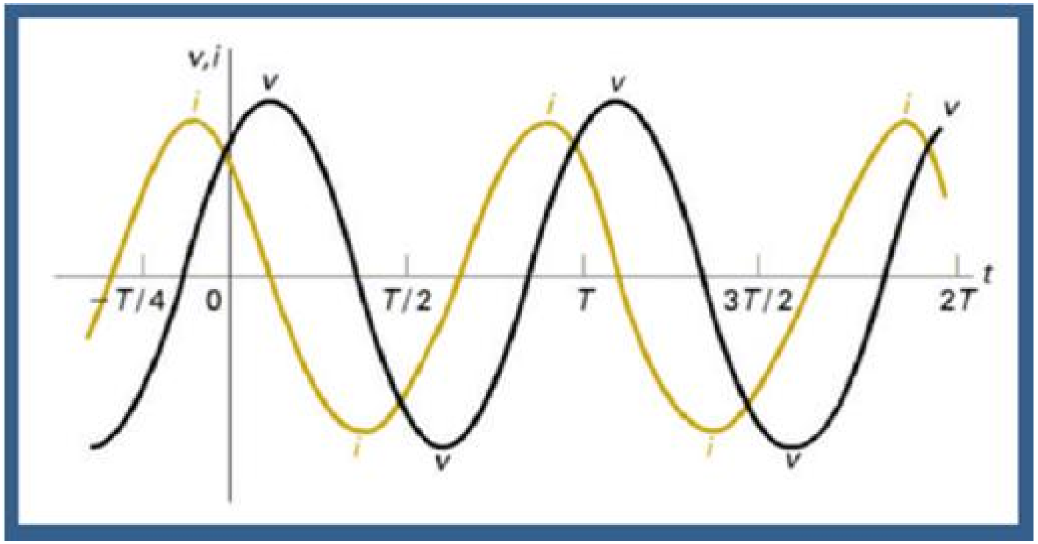




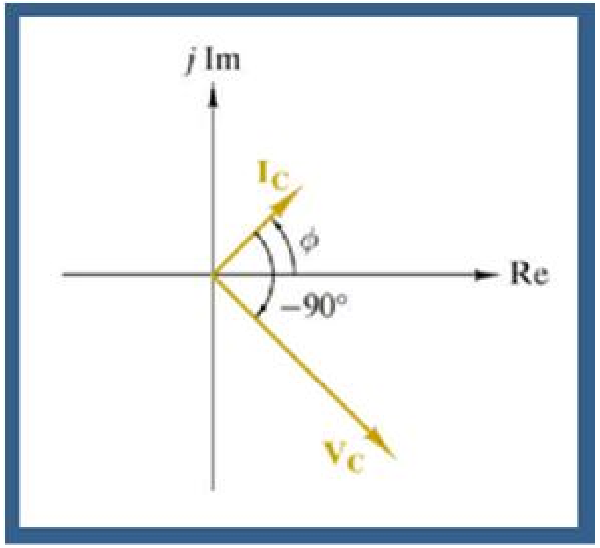


Phasor current of a capacitor LEADS the voltage by 90 degrees.

Phasor Relationships for Capacitor



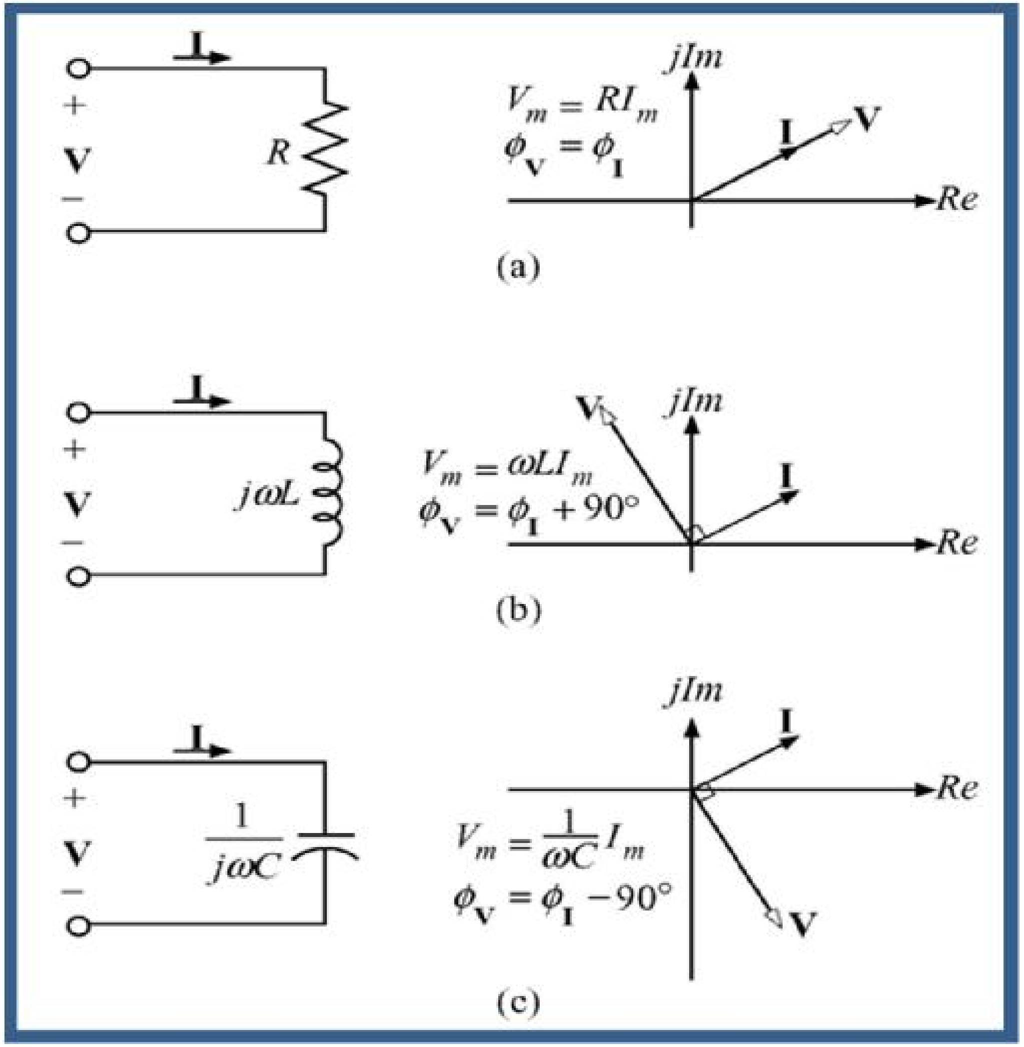
Time Domain



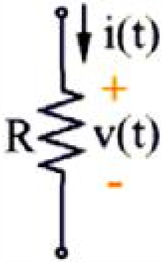
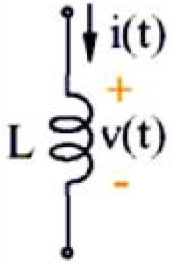
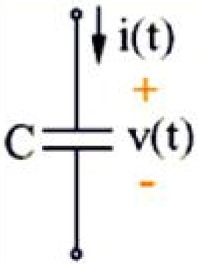
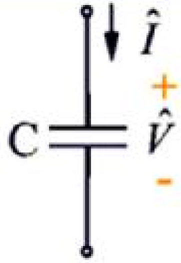
Frequency Domain

Phasor current of a capacitor LEADS the voltage by 90 degrees.

Phasor Relationships for Circuit Elements



Phasor Relationships for Circuit Elements

Time Domain		Frequency Domain	
	$v(t) = Ri(t)$		$\hat{V} = R\hat{I}$ $\hat{I} = G\hat{V}$
	$v(t) = L\frac{di(t)}{dt}$		$\hat{V} = j\omega L\hat{I}$ $\hat{I} = \frac{1}{j\omega L}\hat{V}$
	$i(t) = C\frac{dv(t)}{dt}$ $v(t) = \frac{1}{C}\int i(t)dt$		$\hat{V} = \frac{1}{j\omega C}\hat{I}$ $\hat{I} = j\omega C\hat{V}$